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LETTER TO THE EDITOR

Symmetry breaking by deformations

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Abstract. The effects of symmetry breaking and renormalization on a unified field theory can be simulated by the use of a deformed gauge group instead of $U(3)$. Here the use of a deformation of $u(3)$ called $u_Q(3)$, with three independent parameters, is proposed and illustrated by obtaining a mass formula with electromagnetic mass splitting as well as the hypercharge and isospin dependence for the baryons.

Unification of field theories of the elementary particles based on the symmetries of the gauge groups $SU(3)$ [1–3], or the various grand unifications based on $SU(5)$ [4] and still larger gauge groups [5], require a symmetry-breaking mechanism to account for the observed isospin and charge dependence of mass spectra, decay rates and other branching parameters. Since the known exactly soluble field theories are of restricted physical significance, an exact method of symmetry breaking based on a deformation of the relevant gauge groups is of interest, either in its own right or as a means of simulating the effect of the usual method based on renormalization and the Higgs mechanism [6]. The well known q-deformations [7, 8] of the Lie algebras and superalgebras involve a single parameter and preserve most of the symmetries of the original algebras. However, there are now various instances of deformations [9, 10] that involve more than one parameter and may, therefore, provide a more comprehensive symmetry-breaking mechanism.

In the following, we initiate an approach to symmetry breaking based on a particularly simple deformation of $u(n)$, with up to n distinct parameters $Q_1, \ldots, Q_n$, that we call $u_Q(n)$. In symmetric representations it is the same as the algebra defined by Green [11] with the generalized commutation relations

$$[e_k^j, e_m^l]_w \equiv e_k^j e_m^l - (w_k^l/w_m^j) e_m^l e_k^j = \delta_k^l e_m^j - (w_k^l/w_m^j)\delta_m^j e_k^l$$
$$w_k^j = 1 + (Q_j - 1)\delta_k^j \tag{1}$$

but is here defined so that it has more general irreducible representations. Although the discussion that follows is limited to $u_Q(3)$, it can be generalized under certain conditions to larger gauge groups.

The Q-deformed algebra $u_Q(3)$ corresponding to $u(3)$ has elements e_k^j $(j, k = 1, \ldots, 3)$ satisfying the generalized commutation relations

$$[e_k^j, e_m^l] \equiv e_k^j e_m^l - e_m^l e_k^j = f_k \delta_k^l e_m^j - \delta_m^j e_k^l f_m$$
$$e_k^j f_k = Q_k f_k e_k^j \qquad (j \neq k). \tag{2}$$

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As usual, e_j^k is the Hermitean conjugate of e_k^j. This algebra can obviously be reduced to $u(3)$ when $Q_1 = Q_2 = Q_3 = 1$. From the above relations with $l = m = k$ it follows without difficulty that

$$f_k = (Q_k - 1)e_k^k + 1 = Q_k^{n_k} \tag{3}$$

where the n_k satisfy

$$(n_k + 1)e_k^j = e_k^j n_k \tag{4}$$

and have non-negative integral eigenvalues that will be interpreted subsequently as quark numbers. In symmetric representations

$$e_k^j e_l^k = (Q_k e_k^k + 1)e_l^j \qquad (j \neq k) \tag{5}$$

and the equivalence of (1) and (2) can be demonstrated. In the more general representations, there is no analogue of the polynomial invariants found for the usual q-deformed algebra [12, 13]; however, a highest-weight vector ψ in a tensor representation of $u_Q(3)$ can be defined by

$$e_k^j \psi = (Q_j^{l_j} - 1)\delta_k^j \psi/(Q_j - 1) \qquad (1 \leqslant j \leqslant k \leqslant 3) \tag{6}$$

where (l_1, l_2, l_3) is the set of highest weights that are invariants and serve to label the representation. Since

$$e_k^j e_j^k = e_j^j (Q_k^k e_k^k + 1) \tag{7}$$

is positive definite, $l_1 \geqslant l_2 \geqslant l_3 \geqslant 0$. General vectors of the representation are obtained by multiplying ψ by products of the e_j^k with $j < k$.

Although for general values of the Q_j all the invariants of $u_Q(3)$ or $u_Q(2)$ are transcendental, we may define a Q-variant M as a polynomial in the elements of the algebra of the type

$$M = M_0 Q_1^{\mu_1} Q_2^{\mu_2} Q_3^{\mu_3} \tag{8}$$

where the μ_j are integers that may depend on the invariants of $u_Q(3)$ and its subalgebras. These may include the quark numbers n_j that, according to (2), are invariants of the three subalgebras $u_Q(1)$, and are given by (3). The invariants m_1 and m_2 of the subalgebra $u_Q(2)$ of $u_Q(3)$ are the analogues of l_1, l_2 and l_3 in (6) with $1 \leqslant j \leqslant k \leqslant 2$, and satisfy $l_3 \leqslant m_2 \leqslant l_2 \leqslant m_1 \leqslant l_1$. There is no difference between these invariants and those of the undeformed algebra. A variant of type (8) has the property

$$e_k^j (M/M_j) = (M/M_k) e_k^j \tag{9}$$

where M_j and M_k are either constants or other Q-variants so that (M_j/M_k) can be interpreted as a renormalization factor when the Q_j are regarded as renormalization constants. It is evident from this relation that the M_j can differ at most by a constant multiple from the eigenvalues of the variant M. The simplest Q-variants are the generalized structure coefficients f_k in (2).

The deformation of the $u(3)$ algebra allows us to obtain formulae for mass differences that are somewhat different from, though similar in principle to, those found from the usual symmetry-breaking methods. The number of invariants is, of course, sufficient to fit any empirical mass spectrum exactly, but it is also possible to obtain relatively simple formulae that are Q-variants as defined in (8) and fit the empirical data as well as or better than the existing well known formulae. To simulate the reduction of $u(3)$ to $su(3)$ we may choose $\mu_1 + \mu_2 + \mu_3 = 0$; the variant defined in (8) then depends only on the ratios Q_j/Q_k as parameters and the differences $\mu_j - \mu_k$ of the central invariants.

As an example we give an analogue of the mass formula of Gell-Mann [14] and Okubo [15] for the baryons that is also good for the mass differences of the particles forming isospin multiplets.

To first (linear) approximation, the differences of the μ_j in (8) are given by

$$\mu_2 - \mu_1 = n_2 - n_1 \qquad 2\mu_3 - \mu_1 - \mu_2 = 2n_3 - n_1 - n_2 + m_2 - m_1 \quad (10a)$$

and for a somewhat better (quadratic) approximation

$$\mu_2 - \mu_1 = n_2 - n_1 - n_1 n_2 + 3n_3(n_2 - n_1 - m_2 + m_1)/2$$
$$2\mu_3 - \mu_1 - \mu_2 = 3n_3 - n_1 - n_2 + 3n_3(n_1 + n_2 - m_2 + m_1)/2. \quad (10b)$$

M_0 is an invariant of $u_Q(3)$ and $u(3)$. Thus, the masses of the $u_Q(2)$ multiplets depend on the isospin $m_1 - m_2$ and strangeness n_3, though exponentially instead of linearly on the invariants as in the Gell-Mann–Okubo formula. When the masses are represented by the same symbols as the particles, formula (10b) yield the ratios

$$N/P = (\Sigma^-/\Sigma^0)^{1/3} = (\Sigma^0/\Sigma^+)^{1/2} = (\Xi^-/\Xi^0)^{1/4} = Q_2/Q_1 \approx 1.0013$$
$$(\Lambda^0/P)^{6/7} = (\Sigma^0/\Lambda^0)^2 = (\Xi^-/\Sigma^0)^{3/2} = Q_3^2/Q_1 Q_2 \approx 1.15 \quad (11)$$

all of which are correct to within one digit of the last significant figure. The masses of the baryon decuplet are given in similar fashion by

$$(\Sigma/\Delta)^{6/5} = (\Xi/\Sigma)^{4/3} = (\Omega/\Xi)^{3/2} = Q_3^2/Q_1 Q_2 \quad (12)$$

and the values of the Q_j/Q_k are the same as for the octet, though the numerical constants in (10b) as well as M_0 necessarily depend on the representation.

The formulae (10a) and (10b) both satisfy the requirement that the mass should be expressible as a polynomial in values of the e_j^j. The renormalization constant $Q_3^2/Q_1 Q_2$ reduces the $su(3)$ symmetry to $su(2) \otimes u(1)$, and could be related to a charge-independent quark–gluon interaction, while the mass splitting within isospin multiplets corresponding to the value of $Q_2/Q_1 \approx 1 + \alpha/2\pi$ could well be related to the electromagnetic interaction, though the similarities of the formulae for the differences $\mu_j - \mu_k$ (and the signs of the mass differences) suggest that the effect of the interactions are not independent of one another.

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